

**O. EXPRESII ALGEBRICE - PROBLEME REZOLVATE**

1) Fie expresia:  $E(x) = \left( \frac{x}{x^2 - 9} + \frac{2}{3 + x} + \frac{1}{3 - x} \right) : \frac{2x - 9}{x^2 - 2x - 15}$

a) Determinati valorile lui  $x$  pentru care  $E(x)$  are sens

b) Determinati  $a \in \mathbb{Z}$  pentru care  $E(a) \in \mathbb{Z}$

c) Rezolvati in  $\mathbb{N}$  inecuatia  $(x + 3) \cdot E(x) \leq 0$

**REZOLVARE**

a) Egalez numitorii fractiilor cu 0, iar la fractia care este dupa semnul : egalez si numaratorul cu 0

$$x^2 - 9 = 0 \Rightarrow (x - 3)(x + 3) = 0 \Rightarrow x - 3 = 0 \Rightarrow x = 3$$

$$x + 3 = 0 \Rightarrow x = -3$$

$$x^2 - 2x - 15 = 0 \Rightarrow x^2 - 5x + 3x - 15 = 0 \Rightarrow x(x - 5) + 3(x - 5) = 0 \Rightarrow (x - 5)(x + 3) = 0 \Rightarrow x - 5 = 0 \Rightarrow x = 5$$

$$x + 3 = 0 \Rightarrow x = -3$$

$$2x - 9 = 0 \Rightarrow 2x = 9 \Rightarrow x = \frac{9}{2}$$

Deoarece  $E(x)$  are sens  $\Rightarrow x \in \mathbb{R} - \left\{ -3; 3; \frac{9}{2}; 5 \right\}$

b) Mai intai aduc  $E(x)$  la forma cea mai simpla apoi determin  $E(a)$ , inlocuind in  $E(x)$  pe  $x$  cu  $a$

$$E(x) = \left[ \frac{x}{(x - 3)(x + 3)} + \frac{x - 3}{x + 3} - \frac{x + 3}{x - 3} \right] : \frac{2x - 9}{(x - 5)(x + 3)} = \frac{x + 2x - 6 - x - 3}{(x - 3)(x + 3)} \cdot \frac{(x - 5)(x + 3)}{2x - 9} =$$

$$= \frac{2x - 9}{(x - 3)(x + 3)} \cdot \frac{(x - 5)(x + 3)}{2x - 9} = \frac{x - 5}{x + 3} \Rightarrow E(x) = \frac{x - 5}{x + 3} \Rightarrow E(a) = \frac{a - 5}{a + 3}$$

$E(a) \in \mathbb{Z}$  daca  $\begin{array}{l} a + 3 \mid a + 3 \\ a + 3 \mid a - 5 \end{array} \begin{array}{l} \cdot (-1) \Rightarrow \\ \cdot (+) \end{array} \Rightarrow \begin{array}{l} a + 3 \mid a + 3 \\ a + 3 \mid -a + 5 \end{array} (+)$

$$a + 3 \mid 8 \Rightarrow a + 3 \in D8 \Rightarrow$$

$a + 3 = 1$	$a + 3 = -1$	$a + 3 = 2$	$a + 3 = -2$	$a + 3 = 4$	$a + 3 = -4$	$a + 3 = 8$	$a + 3 = -8$
$a = -2$	$a = -4$	$a = -1$	$a = -5$	$a = 1$	$a = -7$	$a = 5$	$a = -11$

Din conditia de existenta a fractiilor  $\Rightarrow a \neq 5 \Rightarrow a \in \{-11, -7, -5, -4, -2, -1, 1\}$

c)  $(x + 3) \cdot \frac{x - 5}{x + 3} \leq 0 \Rightarrow x - 5 \leq 0 \Rightarrow x \leq 5 \Rightarrow x \in (-\infty; 5]$

Deoarece  $x \in \mathbb{N} \Rightarrow x \in \{0, 1, 2, 3, 4, 5\}$

Din conditiile de existenta a fractiilor  $\Rightarrow x \neq \{3, 5\}$

$\Rightarrow x \in \{0, 1, 2, 4\}$

$$2) \text{ Fie expresia } E(x) = \frac{1}{2} \cdot \left[ \left( \frac{1}{3-x} \cdot \frac{x^2-9}{x^2-x+1} + \frac{1}{x-3} \right) : \frac{10-x}{x^3+1} - 3 \right]$$

a) Determinati valorile lui  $x$  in care  $E(x)$  nu este definita.

$$5 - a$$

b) Verificati daca  $E(a) = \frac{5-a}{a-3}$

c) Determinati  $a \in \mathbb{Z}$ , astfel incat  $E(a) \in \mathbb{N}$

d) Determinati elementele multimii  $A = \{x \in \mathbb{N}^* \mid E(x) \leq -1\}$

### REZOLVARE

$$a) 3 - x = 0 \Rightarrow -x = -3 / (-1) \Rightarrow x = 3$$

$x^3 + 1 = 0$ ; Descompun  $x^3 + 1$  utilizand formula de calcul prescurtat  $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$$x^3 + 1 = (x + 1)(x^2 - x + 1); \Rightarrow (x + 1)(x^2 - x + 1) = 0 \Rightarrow x + 1 = 0 \Rightarrow x = -1$$

$$10 - x = 0 \Rightarrow -x = -10 / (-1) \Rightarrow x = 10$$

$E(x)$  nu este definita pentru  $x \in \{-1, 3, 10\}$

$$b) E(x) = \frac{1}{2} \cdot \left[ \left( \frac{-1}{x-3} \cdot \frac{(x-3)(x+3)}{x^2-x+1} + \frac{1}{x-3} \right) : \frac{10-x}{(x+1)(x^2-x+1)} - 3 \right]$$

$$E(x) = \frac{1}{2} \cdot \left[ \left( \frac{x-3}{x^2-x+1} + \frac{x^2-x+1}{x-3} \right) \cdot \frac{(x+1)(x^2-x+1)}{10-x} - 3 \right]$$

$$E(x) = \frac{1}{2} \cdot \left( \frac{-x^2 - 3x + 3x + 9 + x^2 - x + 1}{(x-3)(x^2-x+1)} \cdot \frac{(x+1)(x^2-x+1)}{10-x} - 3 \right) = \frac{1}{2} \cdot \left( \frac{10-x}{x-3} \cdot \frac{x+1}{10-x} - 3 \right) =$$

$$= \frac{1}{2} \cdot \left( \frac{x+1}{x-3} - 3 \right) = \frac{1}{2} \cdot \frac{x+1-3x+9}{x-3} = \frac{1}{2} \cdot \frac{-2x+10}{x-3} = \frac{1}{2} \cdot \frac{2(5-x)}{x-3} = \frac{5-x}{x-3}$$

$$E(x) = \frac{5-x}{x-3} \Rightarrow E(a) = \frac{5-a}{a-3}$$

c)  $E(a) \in \mathbb{N} \Rightarrow a-3 \mid a-3$

$$a-3 \mid 5-a \Rightarrow a-3 \mid (a-3) + (5-a) \Rightarrow a-3 \mid 2 \Rightarrow a-3 = D2(+)$$

$$a-3 = 1 \Rightarrow a = 4; \quad a-3 = 2 \Rightarrow a = 5 \Rightarrow a \in \{4, 5\}$$

$$d) E(x) \leq -1 \Rightarrow \frac{5-x}{x-3} \leq -1 \Rightarrow \frac{5-x}{x-3} + 1 \leq 0 \Rightarrow \frac{5-x+x-3}{x-3} \leq 0 \Rightarrow \frac{2}{x-3} \leq 0$$

**Fractia este negativa daca numaratorul si numitorul au semne opuse.**

Deoarece  $2 > 0$ , fractia va fi negativa pentru  $x-3 < 0 \Rightarrow x < 3 \Rightarrow x \in (-\infty; 3)$

Deoarece  $x \in \mathbb{N}^* \Rightarrow A = \{1, 2\}$

$$3) \text{ Fie expresia } E(x) = \frac{2x}{x-1} + \frac{4}{2x^2-x} \cdot \left[ \frac{2-x}{1-4x^2} \cdot \frac{1-16x^3+(-4x)+4x}{2x^2+x} - \frac{x+2}{2x^2-3x^2:3x} \right]^{-1}$$

a) Determinati valorile lui  $x \in \mathbb{R}$ , pentru care  $E(x)$  are sens.

b) Verificati daca  $E(x) = \frac{x+1}{x-1}$

c) Determinati  $a \in \mathbb{Z}$  astfel incat  $E(a) \in \mathbb{Z}$

### REZOLVARE

a)  $x-1=0 \Rightarrow x=1$  ;  $2x^2-x=0 \Rightarrow x(2x-1)=0 \Rightarrow x=0$  ;  $2x-1=0 \Rightarrow 2x=1 \Rightarrow x=\frac{1}{2}$

$1-4x^2=0 \Rightarrow (1-2x)(1+2x)=0 \Rightarrow 1-2x=0 \Rightarrow 2x=1 \Rightarrow x=\frac{1}{2}$   
 $\Rightarrow 1+2x=0 \Rightarrow 2x=-1 \Rightarrow x=-\frac{1}{2}$

$E(x)$  are sens daca  $x \in \mathbb{R} \setminus \left\{ -\frac{1}{2}, 0, \frac{1}{2}, 1 \right\}$

b)  $E(x) = \frac{2x}{x-1} + \frac{4}{x(2x-1)} \cdot \left[ \frac{2-x}{(1-2x)(1+2x)} \cdot \frac{1+4x^2+4x}{x(2x+1)} - \frac{x+2}{2x^2-x} \right]^{-1}$

$E(x) = \frac{2x}{x-1} + \frac{4}{x(2x-1)} \cdot \left[ \frac{2-x}{(1-2x)(1+2x)} \cdot \frac{(1+2x)^2}{x(2x+1)} - \frac{x+2}{x(2x-1)} \right]^{-1}$

$E(x) = \frac{2x}{x-1} + \frac{4}{x(2x-1)} \cdot \left[ \frac{x-2}{x(2x-1)} - \frac{x+2}{x(2x-1)} \right]^{-1} = \frac{2x}{x-1} + \frac{4}{x(2x-1)} \cdot \left[ \frac{-4}{x(2x-1)} \right]^{-1}$

$E(x) = \frac{2x}{x-1} + \frac{4}{x(2x-1)} \cdot \frac{x(2x-1)}{(-4)} = \frac{2x}{x-1} - 1 = \frac{2x-x+1}{x-1} = \frac{x+1}{x-1} \Rightarrow E(x) = \frac{x+1}{x-1}$

c)  $E(a) = \frac{a+1}{a-1}$  ;  $E(a) \in \mathbb{Z}$  daca  $a-1 \mid a-1$   
 $a-1 \mid a+1 \Rightarrow a-1 \mid (a+1) - (a-1) \Rightarrow a-1 \mid 2 \Rightarrow a-1 = D2$

$a-1=1$      $a-1=-1$      $a-1=2$      $a-1=-2$   
 $a=2$        $a=0$        $a=3$        $a=-1$

Din conditiile de existenta a fractiilor  $\Rightarrow x \neq 0 \Rightarrow a \neq 0 \Rightarrow E(a) \in \mathbb{Z}$  daca  $a \in \{-1, 2, 3\}$