

EXERCITII SI PROBLEME DE ALGEBRA REZOLVATE

A. OPERATII CU NUMERE REALE

a)

$$1) -(-2) + (-5) - |-5| + |-6| - 8 + 9 = +2 - 5 - 5 + 6 - 8 + 9 = +17 - 18 = -1$$

$$2) (-3) \cdot (-5) - (+8) : (-4) - (-4) \cdot (-2) = +15 - (-2) - (+8) = 15 + 2 - 8 = 17 - 8 = 9$$

$$3) 6 - 6 : 2 \cdot 3 + 3 \cdot 6 : 18 = 6 - 3 \cdot 3 + 18 : 18 = 6 - 9 + 1 = 7 - 9 = -2$$

$$4) 8 + 8 : 6 \cdot 9 - 9 : 6 : 2 \cdot 4 = 8 + \frac{8^2}{6} \cdot 9 - \frac{9^3}{6} : 2 \cdot 4 = 8 + \frac{4}{3} \cdot \frac{9^3}{1} \cdot \frac{3}{2} \cdot \frac{1}{2} \cdot \frac{4}{1} = 8 + 12 - 3 = 17$$

$$5) [3^{28} : 3^{18} - (2^{10})^2 + 6 \cdot 6^9] : (2^{10} \cdot 3^{10} - 8^{20} : 4^{20} + 3^4 : 3^{-6}) =$$

$$= (3^{28-18} - 2^{10 \cdot 2} + 6^{1+9}) : [(2 \cdot 3)^{10} - (8 : 4)^{20} + 3^{4-(-6)}] = (3^{10} - 2^{20} + 6^{10}) : (6^{10} - 2^{20} + 3^{10}) = 1$$

$$6) 2^{300} \cdot 3^{200} - 2^{100} \cdot 6^{200} = (2^3)^{100} \cdot (3^2)^{100} - 2^{100} \cdot (6^2)^{100} = 8^{100} \cdot 9^{100} - 2^{100} \cdot 36^{100} = 72^{100} - 72^{100} = 0$$

$$7) -2^2 - (-2)^2 + (-2)^3 - 1^{1900} - (-1)^{2006} + 2005^0 = -4 - (+4) + (-8) - 1 - (+1) + 1 = -4 - 4 - 8 - 1 - 1 + 1 = -17$$

$$8) 1,25 \cdot 2, (6) - 3 \frac{1}{3} = \frac{1}{3} \cdot \frac{5}{4} - \frac{8^2}{3} = \frac{10}{12} - \frac{10}{3} = \frac{10}{3} - \frac{40}{12} = \frac{10}{3} - \frac{40}{12} = 0$$

$$1,25 = \frac{125^{(25)}}{100} = \frac{5}{4} ; 2, (6) = \frac{26-2}{9} = \frac{24^{(3)}}{9} = \frac{8}{3} ; 3 \frac{1}{3} = \frac{3 \cdot 3 + 1}{3} = \frac{10}{3}$$

$$9) 215, (6) - 215,1(6) = \begin{array}{r} 215,66666666..... \\ - 215,16666666..... \\ \hline 0,50000000..... \end{array} = 0,5$$

$$10) \frac{3}{4} + \frac{3}{4} : 0,1^{-1} : (\frac{37}{40} - 1) = \frac{3}{4} + \frac{3}{4} : (\frac{1}{10})^{-1} : (\frac{37-40}{40}) = \frac{3}{4} + \frac{3}{4} : \frac{10}{1} : \frac{(-3)}{40} =$$

$$= \frac{3}{4} - \frac{3}{4} \cdot \frac{1}{10} \cdot \frac{40}{3} = \frac{3}{4} - \frac{3 \cdot 4}{4 \cdot 1} = \frac{3-4}{4} = -\frac{1}{4}$$

b) Sume

$$1 + 2 + 3 + \dots + n = S_n = \frac{n(n+1)}{2}$$

Exemple:

$$1. 1 + 2 + 3 + \dots + 100 = \frac{100 \cdot 101}{2} = 5050$$

$$2. 2 + 4 + 6 + \dots + 100 = 2 \cdot (1 + 2 + 3 + \dots + 50) = 2 \cdot \frac{50 \cdot 51}{2} = 2550$$

$$3. 10 + 11 + 12 + \dots + 50 = S_{50} - S_9 = \frac{50 \cdot 51}{2} - \frac{9 \cdot 10}{2} = 1230$$

$$4. 1 + 3 + 5 + 7 + \dots + 99 = S_{99} - (2 + 4 + 6 + \dots + 98) = S_{99} - 2 \cdot S_{49} = 99 \cdot 50 - 49 \cdot 50 = 50 \cdot 50 = 2500$$

$$5. \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \dots + \frac{1}{49 \cdot 50} = \frac{3-2}{2 \cdot 3} + \frac{4-3}{3 \cdot 4} + \frac{5-4}{4 \cdot 5} + \dots + \frac{50-49}{49 \cdot 50}$$

$$= \frac{3}{2 \cdot 3} - \frac{2}{2 \cdot 3} + \frac{4}{3 \cdot 4} - \frac{3}{3 \cdot 4} + \frac{5}{4 \cdot 5} - \frac{4}{4 \cdot 5} + \dots + \frac{50}{49 \cdot 50} - \frac{49}{49 \cdot 50} =$$

$$= \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \dots + \frac{1}{49} - \frac{1}{50} = \frac{1}{2} - \frac{1}{50} = \frac{24}{50} = \frac{12}{25}$$

$$6. \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + \frac{1}{90} = \frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{2 \cdot 3 \cdot 4 \cdot 5} + \dots + \frac{1}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 9} = \frac{1}{2} + \frac{1}{10} = \frac{6}{10} = \frac{3}{5}$$

mai departe se rezolva ca exercitiul nr.5

$$7. (1 - \frac{1}{2}) \cdot (1 - \frac{1}{3}) \cdot (1 - \frac{1}{4}) \cdot \dots \cdot (1 - \frac{1}{10}) = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} \cdot \frac{5}{6} \cdot \frac{6}{7} \cdot \frac{7}{8} \cdot \frac{8}{9} \cdot \frac{9}{10} = \frac{1}{10}$$

c) Ultima cifra a unui numar

Un numar care se termina cu cifra 1 ridicat la orice putere are ultima cifra 1 $(\dots 1)^n = \dots 1$

Un numar care se termina cu cifra 5 ridicat la orice putere are ultima cifra 5 $(\dots 5)^n = \dots 5$

Un numar care se termina cu cifra 6 ridicat la orice putere are ultima cifra 6 $(\dots 6)^n = \dots 6$

Un numar care se termina cu cifra 4 : ridicat la putere impara are ultima cifra 4 $4^1 = 4$

ridicat la putere para are ultima cifra 6 $4^2 = 16$

Un numar care se termina cu cifra 9 : ridicat la putere impara are ultima cifra 9 $9^1 = 9$

ridicat la putere para are ultima cifra 1 $9^2 = 81$

Exercitii:

a) $2^{425} = 2 \cdot 2^{424} = 2 \cdot 2^{4 \cdot 106} = 2 \cdot (2^4)^{106} = 2 \cdot 16^{106} = 2 \cdot (\dots\dots\dots 6) = \dots\dots\dots 2$, ultima cifra este **2**

b) $3^{479} = 3 \cdot 3^{478} = 3 \cdot 3^{2 \cdot 239} = 3 \cdot (3^2)^{239} = 3 \cdot 9^{239} = 3 \cdot (\dots\dots\dots 9) = \dots\dots\dots 7$, ultima cifra este **7**

c) Aflarea a n-a zecimala a unui numar zecimal periodic

Exercitii:

1) Aflati a 101 zecimala a numarului 2,34(125) .

Se rezolva parcurgand urmatoarele etape:

* $2,34(125) = 2,34125125125125\dots\dots\dots 125\dots\dots\dots$

* **din 101 se scade numarul de cifre care sunt in afara perioadei** : $101 - 2 = 99$

* **rezultatul care ramine se imparte la numarul de cifre din perioada** : $99 : 3 = 33$ rest 0

* **daca restul este: 0** atunci zecimala 101 este **ultima cifra din perioada**, adica 5

1 atunci zecimala 101 este **prima cifra din perioada**, adica 1

2 atunci zecimala 101 este **a doua cifra din perioada**, adica 2

2) Aflati a 2001 zecimala a numarului 0,134(32156)

$2001 - 3 = 1998$ $1998 : 5 = 399$ rest **3** , zecimala 2001 este **a treia cifra din perioada**, adica **1**

3) Aflati a 205 zecimala a numarului 53 / 7

$53 / 7 = 7,57142857142857 = 7,57(142857)$

$205 - 2 = 203$; $203 : 6 = 33$ rest **5** ; zecimala 205 este **a cincea cifra din perioada** , adica **5**